Study of exponential function toward growth parameters and correlation of small ruminants

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Abstract
This study set out to identify which of the different non-linear growth curve model - Von-Bertalanffy, Gompertz, Richards, Weibull, exponential function, and logistic - was best suited for characterising the growth curve. It included body weight records for 715 female and 340 male kids, recorded every three months from birth to the 12th month of age. Asymptotic live weight ("A") was higher in males than in females. Males had a higher maturity rate ("C") than females. In conclusion, live weight as a function of age for male and female goats and sheep was best estimated using non-linear growth models. Residual SS (SSE), Residual MS (MSE), AICC and R² were calculated as 13.086, 1.1897, 8.7528 and 97.07, respectively.

Growth impacts the different products an animal produces, such as milk, meat, and other foods. It is an important stage in the animal's life. It is better to study animal growth statistically because it is difficult to understand a sequence of weight-age data points analytically. Growth curve inheritance is necessary to comprehend evolutionary change for developing effective breeding approaches.

Keywords: Body weight, goat, sheep

Introduction
Models of the latter include those of von Bertalanffy, Gompertz, Richards, and Verhulst (logistic growth). The results of a Google Scholar search for papers on the application of the Brody model for sheep and goats were roughly 22,500 and 15,500, respectively. The Verhulst model yielded approximately 5500 and 3500 hits, the Gompertz model produced roughly 4000 and 2000 hits, and the von Bertalanffy model produced roughly 2500 and 1500 hits (Brunner and Kühleitner, 2020) [4].

Animal research has conducted numerous studies to characterise the growth patterns of the animals. These studies considered models with sigmoidal (S-shaped) growth curves, the Brody model, and negative exponential growth models in addition to fitting conventional models such simple linear or exponential growth to size-at-age data. A variety of non-linear mathematical functions have been used to define the growth curve for evaluating response to a particular treatment at different hours characterising the Brody model for sheep and goats were roughly 22,500 and 15,500, respectively. The Verhulst model yielded approximately 5500 and 3500 hits, the Gompertz model produced roughly 4000 and 2000 hits, and the von Bertalanffy model produced roughly 2500 and 1500 hits (Brunner and Kühleitner, 2020) [4].

Animal research has conducted numerous studies to characterise the growth patterns of the animals. These studies considered models with sigmoidal (S-shaped) growth curves, the Brody model, and negative exponential growth models in addition to fitting conventional models such simple linear or exponential growth to size-at-age data. A variety of non-linear mathematical functions have been used to define the growth curve for evaluating response to a particular treatment at different times, interaction between and within populations for identifying heavier animals at an early age, and Gompertz (Laird, 1965) [6], Bertalanffy (Bertalanffy, 1957) [5], Logistic (Nelder, 1961) [8], and negative exponential (Bathaei and Leroy, 1998) [9] (Magotra et al., 2021) [7].

Materials and Methods
The sheep and goats were maintained on a vast field grazing method. The goats grazed on the pasture for six to eight hours every day. The trees, bushes, and grasses that are available for the goat are classified as follows: Monsoon (Kair, Dhaman, Dudh, Patharchatta, Motha, Akra, and Thur), winter (Neem, Motha, Akra, Keekar, and Beri), and summer (post-harvest leftover residue of Gramme pea (Chickpea), Babul, Kair, and Khejri).

Statistical Analysis
Body weights were standardized for 30, 60, 120, 150, 210, 240, 300 and 330 days using the following methodology (Warwick and Legates, 1979) [6].

\[ P_t = P_{near} + ADG \ (i - age \ P_{near}) \]
where \( P_i \) is the standardized weight at standard age \((i)\), \( P_{near} \) is the weight nearest to standard age \((i)\), ADG is average daily gain considered among the weights after standard age \((i)\) and before standard age \((i)\) is age to which weight is standardized, and age \( P_{near} \), age to weight nearest to standard age \((i)\) considering. Average daily gain in the body weight of individual was calculated by using the following formula (Brody, 1964) [3].

\[
\text{Average daily gain} = \frac{W_2 - W_1}{t_2 - t_1}
\]

Where: \( W_2 = \) Final body weight (kg); \( W_1 = \) Initial body weight (kg); \( t_2 = \) Age of the animal at the end of the period (days); \( t_1 = \) Age of the animal at the beginning of period (days)

Different mathematical models were used to estimate growth curve parameters using Sistatistics 10 software.

<table>
<thead>
<tr>
<th>Non-linear growth curve models</th>
<th>Equations</th>
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<tr>
<td>Exponential</td>
<td>( A \times (1 - e^{(-C \times \text{TIME})}) )</td>
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Where: \( W_i = \) the expected body weight (Kg) at ‘t’ time; \( A = \) is the asymptotic weight; \( B = \) the folding point of growth; \( K = \) the rate of growth; \( m = \) Shape parameter; \( e = \) random error; \( e = \) the base of natural logarithm; \( t = \) time (birth to 12th month of age)

Residuals were plotted graphically which gave an accuracy of the model to fit the growth curves.

\[
e = y_i - \hat{y}_i
\]

Where: \( Y_i = \) Observed body weight at time “t”; \( \hat{y}_i = \) Predicted body weight by regression model at time “t”.

The Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) was calculated as below (Topal and Balukbasi, 2008) [12].

\[
\text{MAE} = \frac{\sum_i^n |y_i - \hat{y}_i|}{n}
\]

\[
\text{MAPE} = \frac{1}{n} \sum_i^n \frac{|y_i - \hat{y}_i|}{y_i}
\]

Where: \( Y_i = \) Observed body weight at time “t”; \( \hat{y}_i = \) Predicted body weight by regression model at time “t”; \( n = \) Number of observations (data points).

In the case the sample size is smaller than the number of model parameter \((N < K < 40)\), the AIC might not be accurate then after use of Akaike’s Information Criteria (AICC) was appropriate and calculated as using the equation Motulsky and Christopoulos (2004) [10].

\[
\text{AIC}_c = \text{AIC} + \frac{2p(p + 1)}{n - p - 1}
\]

Where: \( \text{AIC} = \) Akaike’s Information Criteria

Therefore, \( \text{AIC}_c \) is a good static for comparison of models of different complexity because it adjust the residual sum of squares (RSS) for number of parameters in the model. A smaller numerical value of AICC indicates a better fit when comparing models.

The Chi-square (\( \chi^2 \)) values is used to designate the relationship between actual and predicted body weights whether there is a significant difference between the predicted and the observed body weights.

\[
\chi^2 = \frac{(y_i - \hat{y}_i)^2}{\hat{y}_i}
\]

Where: \( Y_i = \) Observed body weight at time “t”; \( \hat{y}_i = \) Predicted body weight by regression model at time “t”.

Results and Discussion

Growth curve models were used to estimate the growth curve parameters "A," and "C." R2, R2 adj, Residual SS (SSE), Residual MS (MSE), AICC, and Chi-square (X2) values were used to assess the goodness of fit.

For the male and female in the growth curve model, parameter \( A \) yielded values of 23.82±1.23 and 22.24± kg, respectively. This is the maximum value or quantity that the system can withstand in the long run. In the context of biological populations, it may represent the maximum population that an ecosystem can sustain.

A smaller \( c \) value indicates a more gradual curve, whereas a bigger \( c \) value indicates a steeper curve. It usually has to do with the rate at which the population is approaching the carrying capacity.

For the male and female, respectively, a growth curve model with values for parameter \( C \) of 0.22 and 0.21 was supplied. The population growth rate is set by selecting this option. A higher \( k \) number corresponds to faster growth, while a lower \( k \) value to slower growth.

In practical terms, these parameters are discovered by fitting the growth model to empirical data using statistical techniques. The curve is adjusted to more closely match the observed growth trend of the system under investigation. Growth models are widely used in ecology, biology, and other fields to predict population dynamics and growth phenomena.

The adjusted coefficient of determination \((R_{(\text{Adj.})}^2)\) for all growth curve models was determined to be 96.93 and 97.00 percent, suggesting that the models were all fitted to the body weight of male and female Sirohi goats, respectively. The adjusted coefficient of determination \((R_{(\text{Adj.})}^2)\) for the growth curve model was found to be the best fit.

The residual error (SSE) was estimated using the growth curve model that fit the data the best. The values of 14.83 and 12.73 growth curve model’s residual error (SSE) for male and female, respectively.

The growth curve model was given the residual mean square errors (SSE) for males and females, 1.34 and 1.15, respectively. It is important to keep in mind that the dependent variable and the mean square error (MSE) share the same unit, which makes interpretation easier in the context of the current problem. Nevertheless, depending on the precise objectives and aspects of the modelling task, MSE could not be the sole statistic to take into account. It ought to be applied in concert with additional assessment measures.

The corrected Akaike information criteria (AICC) values for males and females were obtained by the growth curve model with 10.37 and 8.40, respectively. In non-linear growth curve models, the X2 (Chi-square) values between the expected and actual body weights were non-significant \((p \geq 0.01)\), demonstrating that the predicted and observed body weights were not different in Sirohi goats.
According to the goodness of fit statistics for the body weight of the Sirohi goat, the Brody model was determined to be the best fit for explaining the growth pattern in the current inquiry. Von-Bertalanffy, Gompertz, and Logistic growth curve models were shown to be the next best fits. Comparable results were found in the Kambing Katjang goat by Tsukahara et al. (2008) [13], the Young hair goat by Tatar et al. (2009) [15], the Beetal goat by Waheed et al. (2011) [14], and the Repartida goat by Pires et al. (2017) [9].

Conclusion

The study found that a number of factors, including but not limited to flock size, species, breed, selection techniques, environmental and managerial conditions, farmer socioeconomic position, variations in growth curve models, and nutritional traits of breeds and species, contribute to variation in growth curve model parameters.

Declarations of Competing Interest

The authors declare that they have no conflicts of interest associated with this publication.

References